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# Propagation of Error in the Calculation of Airplane Thrust and Drag

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**Knowledge of an airplane's drag polar is needed to establish the performance characteristics of the airplane. In-flight measurements are frequently made during the course of test programs to define drag polars. Inaccuracies in the measured variables can contribute to significant uncertainties in computed lift and drag coefficients. To gain an appreciation of the magnitude of these uncertainties, equations have been derived for an aircraft powered by a nonafterburning turbojet engine which express the inaccuracies in lift and drag coefficients in terms of inaccuracies in measured variables. As a by-product, the variables that are the source of the larger inaccuracies in the drag polar can be found and the need for improved instrumentation identified. Sample calculations were made using data from the Air Force performance tests on the A-37A, and inaccuracies in its drag polar, as well as in the intermediate answers (gross thrust, inlet momentum, and drag), are shown graphically. These calculations indicate that the most critical measurements are of turbine discharge pressure and vane alignment (to orient axes of accelerometers used to compute excess thrust); degradation of computed drag from errors in these variables is most pronounced at high lift coefficients.**

## Nomenclature

$A_9$ = nozzle area, ft²
$C_D$ = drag coefficient, dimensionless
$C_g$ = nozzle coefficient, dimensionless
$C_L$ = lift coefficient, dimensionless
$D$ = drag, lb
$F_e$ = inlet momentum (ram drag), lb
$F_{ex}$ = excess thrust, lb
$F_g$ = gross thrust, lb
$H$ = geopotential altitude, ft
$I(\ )$ = interval estimate of ( )
$K_1$ = $6.87535 \times 10^{-6}$, 1/geopotential ft
$K_2$ = 5.2561
$K_3$ = 17058.8 psf
$K_4$ = 0.43034 ft²/lb
$K_5$ = 661.48 knots
$K_\alpha$ = percentage point
$M$ = Mach number, dimensionless
$n_x$ = load factor along wind x-axis, dimensionless
$n_z$ = load factor along wind z-axis, dimensionless
$P_a$ = ambient pressure, in. Hg
$P_{t_5}$ = turbine discharge pressure, in. Hg
$S$ = wing area, ft²
$T$ = ambient temperature, °K
$V_c$ = calibrated airspeed, knots
$V_t$ = true airspeed, knots
$\Delta V_c$ = compressibility correction $V_e = V_c + \Delta V_c$, knots
$w_a$ = airflow, lb/sec
$W$ = airplane gross weight, lb
$\epsilon$ = misalignment angle, rad
$\rho_{sl}$ = air density at sea level, slugs/ft³
$\sigma$ = standard deviation, air density ratio
$\sigma^2$ = variance

***Superscript***

$*$ = nominal value

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## Introduction

Equations are derived for an aircraft powered by a nonafterburning turbojet engine which show how inaccuracies in variables measured in flight are reflected in an aircraft's performance parameters. With these equations the sensitivity of performance parameters to measured variables can be found. As an example, net thrust is a function of altitude, nozzle discharge pressure, nozzle area, nozzle coefficient, engine airflow, and calibrated airspeed. The sensitivity of net thrust to each of the independent variables can be found. With a knowledge of the inaccuracies of in-flight measurements, the corresponding errors in net thrust can be computed.

Needs for improved instrumentation can then be identified from information of this sort. Also, following the same example, the total error in net thrust can be found which results from the accumulated inaccuracies in all of the independent variables. In this case, a measure of the scatter to be expected in in-flight net thrust (or other parameter) can be computed from tolerances in the individual measurements.

The equations derived describe the propagation of errors in the calculation of gross thrust, inlet momentum, excess thrust, and lift and drag coefficients. To give an appreciation for the magnitude of errors that may be expected in flight data, sample calculations were made using information from the Category II performance tests on the A-37A. Inaccuracies in each of the above parameters are shown in graphical form.

## Propagation of Errors in Flight Test Parameters

Flight test parameters are nonlinear functions of measured in-flight variables. To evaluate the error (tolerance) in a flight test parameter it is necessary to approximate the function by a linear function in the region of interest. This is done by expanding the function into a Taylor series around the test or nominal conditions. Equations are derived which relate tolerances in lift and drag coefficients to tolerances in measured parameters (altitude, calibrated airspeed, EPR, etc.). The propagation of error equations are dependent on the assumption that instrumentation errors are normally distributed. Empirical evidence exists to substantiate this assumption. The propagation of error in gross thrust is carried through in detail.

### Gross Thrust

The following derivations are limited to subcritical flow in the nozzle and to flight below the stratosphere. Assuming $\gamma = 1.33$, we have the usual equation for gross thrust (see Ref. 1, for example);

$$F_g = 570.13 P_a A_8 C_g [(P_{t5}/P_a)^{0.2481} - 1] \quad (1)$$

Atmospheric pressure and geopotential altitude in the troposphere are related by

$$P_a = P_{asl}(1 - K_1 H)^{K_2} \quad (2)$$

where $K_1 = 6.87535 \times 10^{-6}$ geopotential ft$^{-1}$ and $K_2 = 5.2561$. Substituting Eq. (2) in Eq. (1) with $K_3 = 570.13\ P_{asl}$ and $K_4 = (1/P_{asl})^{0.2481} = 0.43034$,

$$F_g = K_3(1 - K_1 H)^{K_2} A_8 C_g K_4 P_{t5}^{0.2481}(1 - K_1 H)^{-0.2481 K_2} - K_3(1 - K_1 H)^{K_2} A_8 C_g \quad (3)$$

Following the procedure described in Ref. 2, the interval estimate of gross thrust is found by first expanding in a Taylor series about the nominal values (denoted by *) and neglecting the higher-order terms;

$$F_g = F_g^* + \frac{\partial F_g}{\partial H}(H - H^*) + \frac{\partial F_g}{\partial A_8}(A_8 - A_8^*) + \frac{\partial F_g}{\partial C_g}(C_g - C_g^*) + \frac{\partial F_g}{\partial P_{t5}}(P_{t5} - P_{t5}^*) \quad (4)$$

The partial derivatives are

$$\partial F_g/\partial H = -0.7519 K_1 K_2 K_3 K_4 A_8 C_g P_{t5}^{0.2481}(1 - K_1 H)^{0.7519 K_2 - 1} + K_1 K_2 K_3 A_8 C_g (1 - K_1 H)^{K_2 - 1}$$

$$\partial F_g/\partial A_8 = K_3 K_4 (1 - K_1 H)^{0.7519 K_2} C_g P_{t5}^{0.2481} - K_3(1 - K_1 H)^{K_2} C_g$$

$$\partial F_g/\partial C_g = K_3 K_4 (1 - K_1 H)^{0.7519 K_2} A_8 P_{t5}^{0.2481} - K_3(1 - K_1 H)^{K_2} A_8$$

$$\partial F_g/\partial P_{t5} = 0.2481 K_3 K_4 (1 - K_1 H)^{0.7519 K_2} A_8 C_g P_{t5}^{-0.7519}$$

Substituting the partial derivatives, after evaluating the constants, in Eq. (4),

$$F_g = F_g^* + [-0.19947 A_8 C_g P_{t5}^{0.2481}(1 - 6.87535 \times 10^{-6} H)^{2.9521} + 0.61646 A_8 C_g (1 - 6.87535 \times 10^{-6} H)^{4.2561}](H - H^*) + [7341.1(1 - 6.87535 \times 10^{-6} H)^{3.9521} C_g P_{t5}^{0.2481} - 17058.8(1 - 6.87535 \times 10^{-6} H)^{5.2561} C_g](A_8 - A_8^*) + [7341.1(1 - 6.87535 \times 10^{-6} H)^{3.9521} A_8 P_{t5}^{0.2481} - 17058.8(1 - 6.87535 \times 10^{-6} H)^{5.2561} A_8](C_g - C_g^*) + 1821.3(1 - 6.87535 \times 10^{-6} H)^{3.9521} A_8 C_g P_{t5}^{-0.7519} \times (P_{t5} - P_{t5}^*) \quad (5)$$

If the independent variables are normally distributed, then $F_g$ is assumed to be normally distributed with a mean of its nominal value, $F_g^*$.

Since Eq. (5) is composed of a linear combination of independent random variables, the variance in gross thrust can be set equal to the sum of the products of variances and squared constants;

$$\sigma_{F_g}^2 = [-0.19947 A_8 C_g P_{t5}^{0.2481}(1 - 6.87535 \times 10^{-6} H)^{2.9521} + 0.61646 A_8 C_g (1 - 6.87535 \times 10^{-6} H)^{4.2561}]^2 \sigma_H^2 + [7341.1(1 - 6.87535 \times 10^{-6} H)^{3.9521} C_g P_{t5}^{0.2481} - 17058.8(1 - 6.87535 \times 10^{-6} H)^{5.2561} C_g]^2 \sigma_{A_8}^2 + [7341.1(1 - 6.87535 \times 10^{-6} H)^{3.9521} A_8 P_{t5}^{0.2481} - 17058.8(1 - 6.87535 \times 10^{-6} H)^{5.2561} A_8]^2 \sigma_{C_g}^2 + [1821.3(1 - 6.87535 \times 10^{-6} H)^{3.9521} A_8 C_g P_{t5}^{-0.7519}]^2 \sigma_{P_{t5}}^2 \quad (6)$$

The variance in altitude can be stated as

$$\sigma_H^2 = [(H - H^*)/K_{\alpha/2}]^2$$

and similarly for the other independent variables. The inaccuracy in altitude $H - H^*$ is associated with the choice of percentage points, $-K_{\alpha/2}$ and $K_{\alpha/2}$. The same percentage points must be used to define inaccuracies in the other variables, $A_8$, $C_g$, and $P_{t5}$. Thus if, for example, the inaccuracies represent a $1\sigma$ deviation, the error in gross thrust which we will compute will also be representative of a $1\sigma$ deviation. The interval estimate for gross thrust $I(F_g)$ is

$$(F_g^* - K_{\alpha/2}\sigma_{F_g}, F_g^* + K_{\alpha/2}\sigma_{F_g}) \equiv I(F_g)$$

Rewriting Eq. (6) and incorporating it in the preceding expression, we have finally

$$I(F_g) = F_g^* \pm \{[-0.19947 A_8 C_g P_{t5}^{0.2481}(1 - 6.87535 \times 10^{-6} H)^{2.9521} + 0.61646 A_8 C_g (1 - 6.87535 \times 10^{-6} H)^{4.2561}]^2 (H - H^*)^2 + [7341.1(1 - 6.87535 \times 10^{-6} H)^{3.9521} C_g P_{t5}^{0.2481} - 17058.8(1 - 6.87535 \times 10^{-6} H)^{5.2561} C_g]^2 (A_8 - A_8^*)^2 + [7341.1(1 - 6.87535 + 10^{-6} H)^{3.9521} \times A_8 P_{t5}^{0.2481} - 17058.8(1 - 6.87535 \times 10^{-6} H)^{5.2561} A_8]^2 \times (C_g - C_g^*)^2 + [1821.3(1 - 6.87535 \times 10^{-6} H)^{3.9521} \times A_8 C_g P_{t5}^{-0.7519}]^2 (P_{t5} - P_{t5}^*)^2\}^{1/2} \quad (7)$$

### Inlet Momentum

The basic equation

$$F_e = 2.0458 w_a M T^{1/2} \quad (8)$$

is used. Airflows are generally gotten from engine manufacturer's charts so that errors in airflow are brought about by both errors in these charts and errors in observed engine speeds. Since errors in the airflow charts will usually be unknown, no attempt was made to separate the two sources of error in the following equations. Rather they are treated as one combined error.

Expanding Eq. (8) in a Taylor series

$$F_e = F_e^* + \frac{\partial F_e}{\partial w_a}(w_a - w_a^*) + \frac{\partial F_e}{\partial M}(M - M^*) + \frac{\partial F_e}{\partial T}(T - T^*) \quad (9)$$

The partial derivatives are

$$\partial F_e/\partial w_a = 2.0458 M T^{1/2}$$

$$\partial F_e/\partial M = 2.0458 w_a T^{1/2}$$

$$\partial F_e/\partial T = 1.0229 w_a M/T^{1/2}$$

Substituting in Eq. (9)

$$F_e = F_e^* + 2.0458 M T^{1/2}(w_a - w_a^*) + 2.0458 w_a T^{1/2}(M - M^*) + (1.0229 w_a M/T^{1/2})(T - T^*) \quad (10)$$

Following the derivation of errors in gross thrust the variance in inlet momentum is

$$\sigma_{F_e}^2 = 4.1853 M^2 T \sigma_{w_a}^2 + 4.1853 w_a T \sigma_M^2 + (1.0463 w_a^2 M^2/T)\sigma_T^2 \quad (11)$$

Errors in observed values of calibrated airspeed $V_c$ and altitude $H$ are preferred, however, to errors in Mach number. These can be related through the equation†

$$\sigma_M^2 = [1/K_5^2\sigma(1 - K_1H)]\sigma_{V_c}^2 + [2.12805K_1(V_c - \Delta V_c)(1 - K_1H)^{2.7561}/K_5\sigma^{3/2} - K_1(V_c - \Delta V_c)/K_5\sigma^{1/2}(1 - K_1H)^{3/2}]^2\sigma_H^2 \quad (12)$$

where $K_1 = 6.87535 \times 10^{-6}$ and $K_5 = 38.967(T_{sl})^{1/2}$. Substituting Eq. (12) in Eq. (11),

$$\sigma_{F_e}^2 = 4.1853 M^2 T \sigma_{w_a}^2 + 4.1853 w_a^2 T\{\sigma_{V_c}^2/K_5^2\sigma(1 + K_1H) + [2.12805K_1(V_c - \Delta V_c)(1 - K_1H)^{2.7561}/K_5\sigma^{3/2} - K_1(V_c - \Delta V_c)/K_5\sigma^{1/2}(1 - K_1H)^{3/2}]^2\sigma_H^2\} + (1.0463 w_a^2 M^2/T)\sigma_T^2 \quad (13)$$

The interval estimate for inlet momentum is

$$I(F_e) = F_e^* \pm \Big(4.1853 M^2 T(w_a - w_a^*)^2 + 4.1853 w_a T\{(V_c - V_c^*)^2/K_5^2\sigma(1 - K_1H) + [2.12805K_1V_e(1 - K_1H)^{2.7561}/K_5\sigma^{3/2} - K_1V_e/K_5\sigma^{1/2}(1 - K_1H)^{3/2}]^2(H - H^*)^2\} + (1.0463 w_a^2 M^2/T)(T - T^*)^2\Big)^{1/2} \quad (14)$$

### Excess Thrust

Errors in excess thrust have been worked out for the case in which excess thrust is computed from flight path acceleration $n_x$, and normal acceleration $n_z$ obtained from vane mounted accelerometers. A misalignment angle $\epsilon$ caused by mislocation of the sensitive axes of the accelerometers and/or upwash is taken into account. Excess thrust is, then

$$F_{ex} = W(n_x \cos\epsilon - n_z \sin\epsilon) \quad (15)$$

Proceeding as before

$$\sigma_{F_{ex}}^2 = (n_x \cos\epsilon - n_z \sin\epsilon)^2\sigma_W^2 + (W\cos\epsilon)^2\sigma_{n_x}^2 + (W\sin\epsilon)^2\sigma_{n_z}^2 + [W(n_x\sin\epsilon + n_z\cos\epsilon)]^2\sigma_\epsilon^2 \quad (16)$$

† $\sigma$ is the air density ratio.

The interval estimate for excess thrust becomes

$$I(F_{ex}) = F_{ex}^* \pm \{(n_x\cos\epsilon - n_z\sin\epsilon)^2(W - W^*)^2 + (W\cos\epsilon)^2(n_x - n_x^*)^2 + (W\sin\epsilon)^2(n_z - n_z^*)^2 + [W(n_x\sin\epsilon + n_z\cos\epsilon)]^2(\epsilon - \epsilon^*)^2\}^{1/2} \quad (17)$$

### Drag

Defining drag as

$$D = F_g - F_e - F_{ex} \quad (18)$$

we may write

$$\sigma_D^2 = \sigma_{F_g}^2 + \sigma_{F_e}^2 + \sigma_{F_{ex}}^2 \quad (19)$$

and

$$I(D) = D^* \pm K_{\alpha/2}(\sigma_{F_g}^2 + \sigma_{F_e}^2 + \sigma_{F_{ex}}^2)^{1/2} \quad (20)$$

### Drag Coefficient

Expanding the equation

$$C_D = 2D/(\rho_{sl}\sigma V_t^2 S) \quad (21)$$

as before, assuming $S$ is constant, we arrive at

$$C_D = C_D^* + (2/\rho_{sl}\sigma V_t^2 S)(D - D^*) - (2D/\rho_{sl}\sigma^2 V_t^2 S)(\sigma - \sigma^*) - (4D/\rho_{sl}\sigma V_t^3 S)(V_t - V_t^*) \quad (22)$$

We wish, however, to change the variables $D$, $\sigma$, and $V_t$ so that errors in $C_D$ are directly related to measured in-flight parameters. First, below the stratosphere

$$\sigma = (1 - K_1H)^{4.2561} \quad (23)$$

Differentiating, we can write

$$\sigma - \sigma^* = -4.2561K_1(1 - K_1H)^{3.2561}(H - H^*) \quad (24)$$

Next it can be shown that

$$\sigma_{V_t}^2 = (1/\sigma)\sigma_{V_c}^2 + [14.6311 \times 10^{-6}(V_e/\sigma^{3/2})(1 - K_1H)^{3.2561}]^2\sigma_H^2 \quad (25)$$

From Eqs. (19), (22), (24), and (25), we can produce

$$\sigma_{C_D}^2 = (2/\rho_{sl}\sigma V_t^2 S)^2(\sigma_{F_g}^2 + \sigma_{F_e}^2 + \sigma_{F_{ex}}^2) + [(8.5122D/\rho_{sl}\sigma^2 V_t^2 S)(K_1)(1 - K_1H)^{3.2561}]^2\sigma_H^2 + (4D/\rho_{sl}\sigma V_t^3 S)^2\{(1/\sigma)\sigma_{V_c}^2 + [14.6311 \times 10^{-6}(V_e/\sigma^{3/2})(1 - K_1H)^{3.2561}]^2\sigma_H^2\} \quad (26)$$

and

$$I(C_D) = C_D^* \pm \Big((2/\rho_{sl}\sigma V_t^2 S)^2[K_{\alpha/2}^2(\sigma_{F_g}^2 + \sigma_{F_e}^2 + \sigma_{F_{ex}}^2)] + [(8.5122D/\rho_{sl}\sigma^2 V_t^2 S) \times (K_1)(1 - K_1H)^{3.2561}]^2(H - H^*)^2 + (4D/\rho_{sl}\sigma V_t^3 S)^2\{(1/\sigma)(V_c - V_c^*)^2 + [14.6311 \times 10^{-6}(V_e/\sigma^{3/2})(1 - K_1H)^{3.2561}]^2(H - H^*)^2\}\Big)^{1/2} \quad (27)$$

### Lift Coefficient

Expanding the equation

$$C_L = 2n_z W/\rho_{sl}\sigma V_t^2 S \quad (28)$$

as for the derivation of error in drag coefficient, we have after substitution of partial derivatives

$$C_L = C_L^* + (2W/\rho_{sl}\sigma V_t^2 S)(n_z - n_z^*) + (2n_z/\rho_{sl}\sigma V_t^2 S)(W - W^*) - (2n_z W/\rho_{sl}\sigma^2 V_t^2 S)(\sigma - \sigma^*) - (4n_z W/\rho_{sl}\sigma V_t^3 S)(V_t - V_t^*) \quad (29)$$

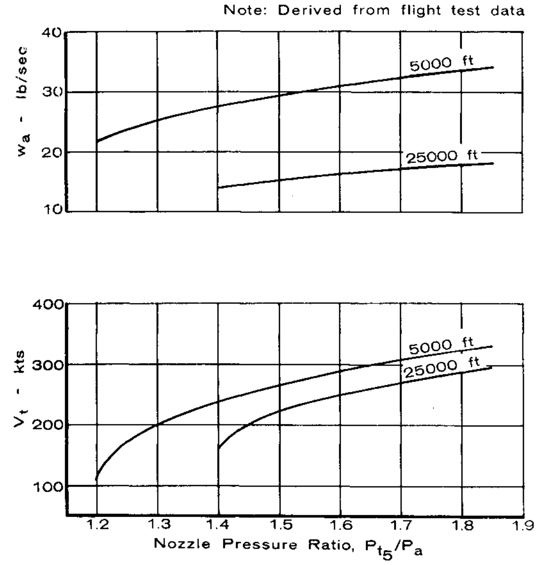


**Fig. 1 Air flow and true speed characteristics.**

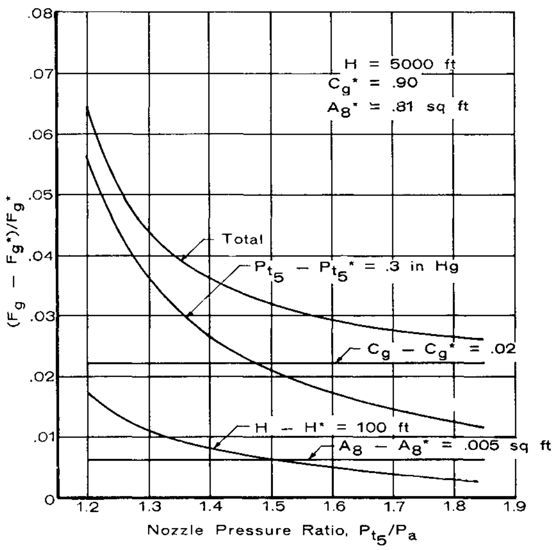


**Fig. 2a Inaccuracy in gross thrust—5000 ft.**

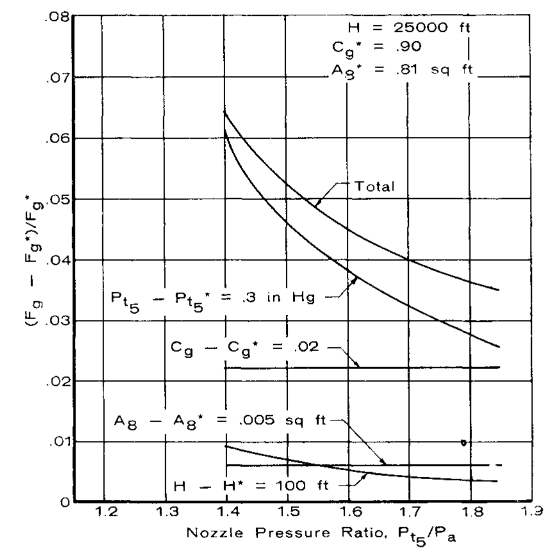


**Fig. 2b Inaccuracy in gross thrust—25,000 ft.**

After substituting Eqs. (24) and (25) the variance can be expressed as

$$\sigma_{C_L}^2 = (2W/\rho_{s1}\sigma V_t^2S)^2\sigma_{n_z}^2 + (2n_z/\rho_{s1}\sigma V_t^2S)^2\sigma_W^2 + [(8.5122n_zW/\rho_{s1}\sigma^2V_t^2S)(K_1)(1 - K_1H)^{3.2561}]^2\sigma_H^2 + (4n_zW/\rho_{s1}\sigma V_t^3S)^2\{(1/\sigma)\sigma_{V_c}^2 + [14.6311 \times 10^{-6}(V_e/\sigma^{3/2})(1 - K_1H)^{3.2561}]^2\sigma_H^2\} \quad (30)$$

and the interval estimate as

$$I(C_L) = C_L^* \pm ((2W/\rho_{s1}\sigma V_t^2S)^2(n_z - n_z^*)^2 + (2n_z/\rho_{s1}\sigma V_t^2S)^2(W - W^*)^2 + [(8.5122n_zW/\rho_{s1}\sigma^2V_t^2S)(K_1)(1 - K_1H)^{3.2561}]^2(H - H^*)^2 + (4n_zW/\rho_{s1}\sigma V_t^3S)^2\{(1/\sigma)(V_c - V_c^*)^2 + [14.6311 \times 10^{-6}(V_e/\sigma^{3/2})(1 - K_1H)^{3.2561}]^2(H - H^*)^2\})^{1/2} \quad (31)$$

## Analysis of A-37A Data

To give an indication of the magnitude of the errors brought about by the propagation of errors in measured values, data representative of the A-37 were processed through the equations in the preceding section. Results are shown in Figs. 2–7 and are typical of other aircraft provided the same percentage errors in $w_a$, $A_8$, etc., are chosen. Nominal values taken

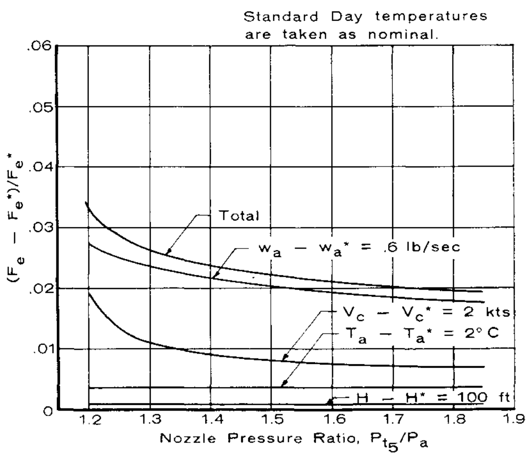


**Fig. 3a Inaccuracy in inlet momentum—5000 ft.**

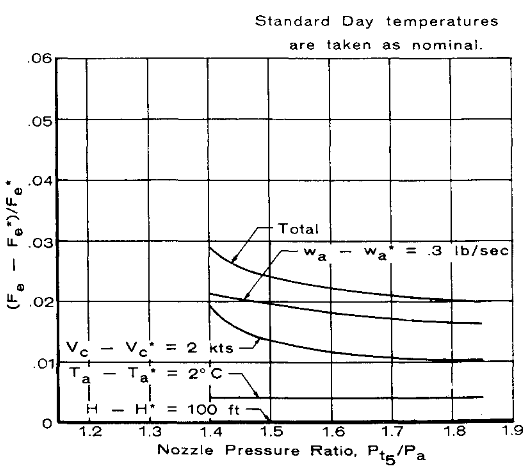


**Fig. 3b Inaccuracy in inlet momentum—25,000 ft.**

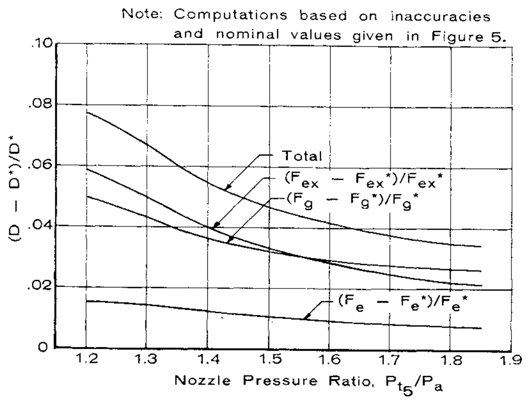


**Fig. 4a Inaccuracy in drag—5000 ft.**

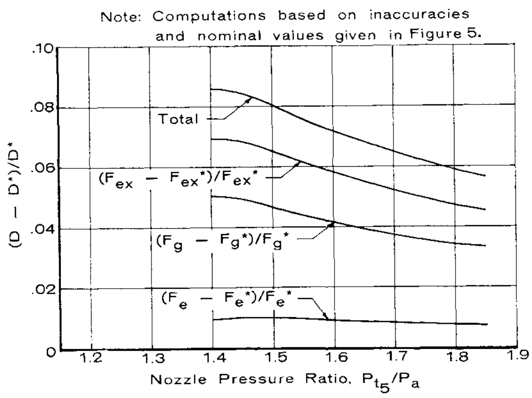


**Fig. 4b Inaccuracy in drag—25,000 ft.**

from Category II tests on the A-37A, S/N 67-14507 were

$$A_8^* = 0.81 \text{ ft}^2, C_g^* = 0.90$$

$$W^* = 9000 \text{ lb}, H^* = 5000 \text{ and } 25000 \text{ ft}$$

$$n_x^* = 0, n_z^* = 1 \text{ g}, \epsilon^* = 1 \text{ deg}$$

Both airflow and true airspeed as a function of engine pressure ratio were derived from flight data (Fig. 1).

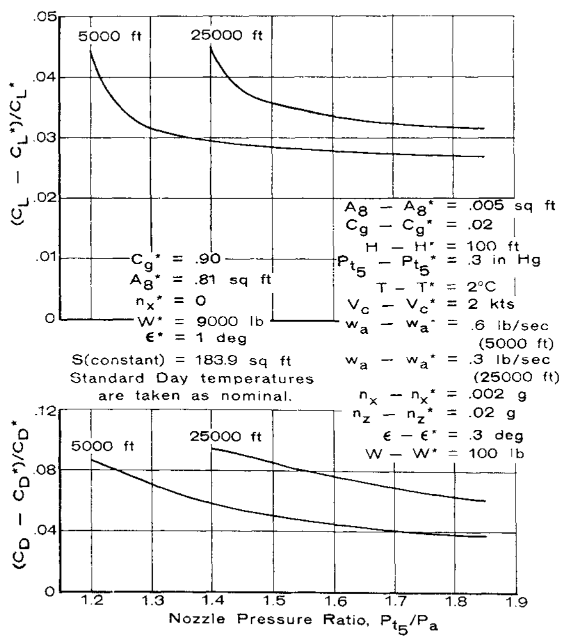


**Fig. 5 Inaccuracy in lift and drag coefficients.**

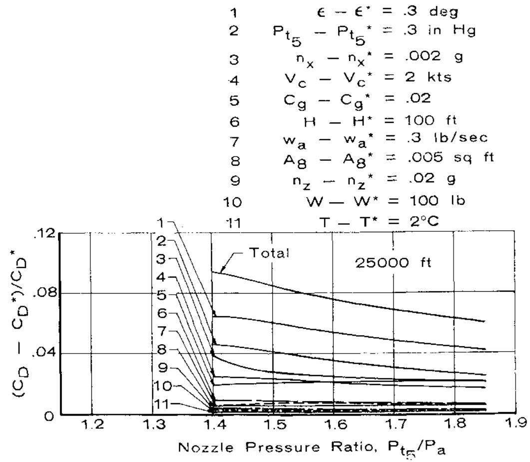


**Fig. 6 Inaccuracy in drag coefficient.**

The inaccuracies assigned to the flight variables are "best estimates" of 1 $\sigma$ deviations and should be considered quite tentative. In the case of airflow, no information on which to base an estimate was available and an arbitrary figure of about 2% was selected. As improved knowledge of the accuracy of flight data becomes available the inaccuracies used in this analysis (see below) can be updated.

$$H - H^* = 100 \text{ ft}, A_8 - A_8^* = 0.005 \text{ ft}^2$$

$$C_g - C_g^* = 0.02, P_{t_5} - P_{t_5}^* = 0.3 \text{ in. Hg}$$

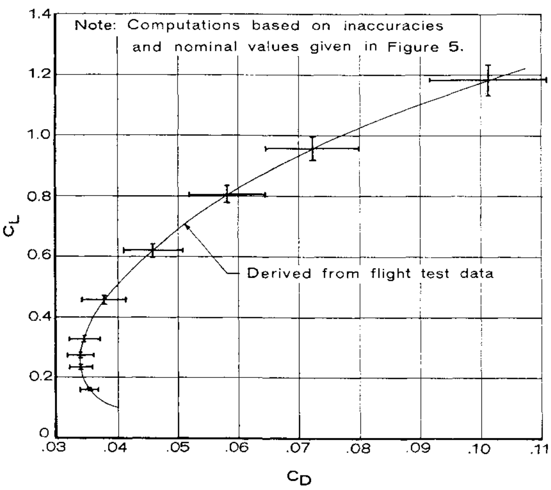


**Fig. 7a Inaccuracy in drag polar—5000 ft.**

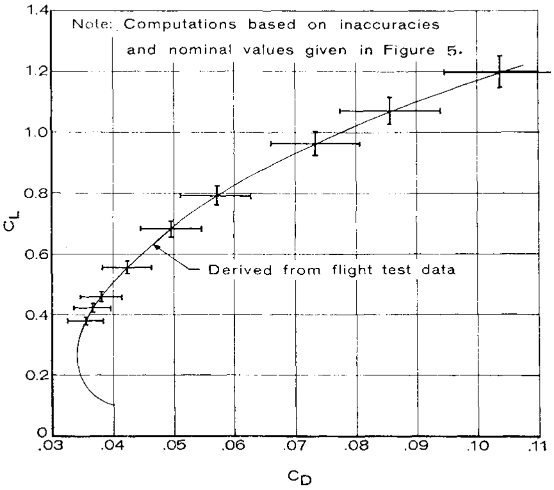


**Fig. 7b Inaccuracy in drag polar—25,000 ft.**

$$w_a - w_a^* = 0.6 \text{ lb/sec at 5000 ft and 0.3 lb/sec at 25000 ft}$$

$$V_c - V_c^* = 2 \text{ knots}, T - T^* = 2° \text{ C}$$

$$W - W^* = 100 \text{ lb}, n_x - n_x^* = 0.002 \text{ g}$$

$$n_z - n_z^* = 0.02 \text{ g}, \epsilon - \epsilon^* = 0.3 \text{ deg}$$

The equations derived in the preceding section take into account excess thrust (as was the case during the A-37A program) computed from a normal accelerometer mounted on a vane which is subject to an error in alignment of $\epsilon$. The misalignment angle $\epsilon$ is taken to be the difference between the sensitive axis of the longitudinal accelerometer and the airplane's velocity vector.

The propagation of error equations derived in the preceding section relate inaccuracies in quantities measured in flight to inaccuracies in gross thrust, inlet momentum, excess thrust, drag (thrust required), and lift and drag coefficients. A program to solve these equations was prepared for the 1620 digital computer. Errors in A-37A data computed with this program are presented in Figs. 2–7 and are discussed below.

From Eq. (1)

$$F_g = f(P_a, A_8, C_g, P_{t5}/P_a)$$

Errors in $F_g$ are shown in Figs. 2a and 2b. Since $A_8$ and $C_g$ are related linearly to $F_g$, a 1% error in either causes a 1% error in $F_g$. An examination of acceleration and deceleration data from the A-37A tests indicates that a $1\sigma$ deviation of 0.02 in $C_g$ is reasonable. Errors in altitude tend to be compensating and are not significant except at very low nozzle pressure ratios. $F_g$ is very sensitive to variation in $P_{t5}$ at low nozzle pressure ratios but becomes much less sensitive as nozzle pressure ratios are increased to moderate values.

By rearranging Eq. (8) in terms of measured quantities, the functional statement

$$F_e = f(w_a, T, V_c, H)$$

can be written. From Figs. 3a and 3b, it can be seen that $F_e$ is rather insensitive to both altitude and temperature variation. Errors in $V_c$ are somewhat more important, particularly at the lower nozzle pressure ratios, but the dominant error in $F_e$ is due to the error in airflow of approximately 2%.

The errors shown in Figs. 2 and 3 are combined with errors in excess thrust to show errors in drag in Fig. 4 and in corresponding drag coefficients in Fig. 5. Also shown in Fig. 5 are errors in lift coefficient brought about by errors in $n_z$, $W$, $V_c$, and $H$. Of these, only errors in $H$ produce insignificant errors in $C_L$. The comparative importance of each of the errors in the variables which affect drag coefficient may be seen in Fig. 6.

Errors in both $C_L$ and $C_D$ appear in Fig. 7 as deviations from the nominal drag polar. Dispersion in $C_L - C_D$ data obtained during flight tests on the A-37A agrees very well with the estimates shown in Fig. 7.

To review the sources in error appearing in the drag polars, relatively minor errors can be expected from measurements of $H$ and $T$; that of $V_c$ is somewhat more important. $A_8$ is probably well enough known, particularly for fixed nozzles, for its contribution to the total error in $C_D$ to be relatively small. Measurements of load factors with sensitive accelerometers mounted on a vane are quite good, but the calculation of excess thrust and hence drag is degraded by uncertainties in the alignment of the vane due to upwash, particularly at low power settings. Uncertainties in $w_a$ from engine manufacturers' charts are not known, but it appears that errors in drag from them are fairly small. Based on thrust stand data, computed values of $C_g$ lead to sizeable errors in drag. Measurement of $P_{t5}$ is critical at low power settings, becoming less important as power setting is increased.

## References

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